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PRELIMINARY STUDY OF MODULATION SYSTEMS FOR SATELLITE COMMUNICATION

Report No. 6 on
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SUMMARY

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A comparative study of power and bandwidth requirements for SSB, FM, and PCM modulation systems for multiple-access satellite communications is given. It is shown how a comparison of information efficiencies can be utilized. These efficiencies are a comparison of each modulation system with SSB which was taken as 100%. Conclusions are that frequency division multiplexing is most advantageous and that FM is somewhat superior to PCM for the S/N ratios required, even neglecting practical difficulties. One other result is that system bandwidth increases with the number of channels, whereas peak power requirements are practically constant for one to 150 channels in SSB and FM systems.

Author

I. INTRODUCTION

This report covers the modulation system aspect of a broader study of multiple-access satellite communication systems. Its objective is to compare the probable suitability of the several attractive or widely discussed modulation methods for those satellite communication systems which permit simultaneous intercommunication between several (or perhaps many) earth stations sharing use of a common satellite repeater. Major attention will be devoted to the analysis and comparison of single sideband modulation (SSB), frequency modulation (FM), and pulse code modulation (PCM) for reasons which will later be made evident. Actually, certain of the modulation comparisons need not be restricted to use in multiple-access systems.

Even though one "random" multiple-access system is envisaged for this report, it is pointed out that the final over-all situation might include many point-to-point trunk route systems in conjunction. In fact, a future problem may well be the solution for a multiple-access configuration between a variety of separate satellite communication networks and the modulation system applicable thereto.

The controversy of active versus passive satellite repeaters has been practically resolved in favor of the active system as applied to world-wide nonmilitary communication systems. Rationale for this conclusion is based upon economics and recent technological advances such as Telstar which show the feasibility of reliable active repeaters. For the passive case, unless the area gain of the repeater equals the repeater gain plus antenna gains of the active system, an inordinate burden is placed on the earth station equipment. For reasons of this kind, this report envisages only active satellite repeaters.

One of the major purposes of this report is to bring out the effects on the modulation system resulting from a multiple-access configuration. Another purpose is to present additional calculations regarding power and bandwidth requirements for various modulation schemes for a specific system. It is intended that the calculations can be easily modified to include various items mentioned under Miscellaneous Aspects as well as different system specifications from those assumed here.

II. OVER-ALL SPECIFICATIONS

Perhaps the most critical aspect of this report is the determination of essential features to be incorporated. Along this line, the following assumptions are given with a brief rationale for each.

(a) A complete global system is envisaged which incorporates multiple access in the satellite repeater. It appears as though technological growth from a world-wide standpoint will be so rapid that some communication will be required from any one area of the earth to all other areas. For this purpose, multiple access would be at least the most straightforward solution as well as providing a backbone system for long term future demands.

(b) Frequency sharing between satellite and ground microwave relay stations is mandatory. From an engineering judgment point of view it appears as though communication satellites will have to operate in the already crowded 1 to 10 Gc region for a rather long period of time. It is pointed out that operation in the 15 to 20 Gc region was the subject of another report.* However since initial systems, at least, will operate at lower frequencies, the most logical choice is to utilize the same or nearly the same bands as the ground microwave relays. This specification will not effect the modulation scheme as long as antenna gain plus attenuation losses remain fixed.

(c) Bandwidth of the over-all system must be conserved, at least eventually. This point is rather obvious, but is spelled out to emphasize that both up-and-down links must be considered. Even if minimum bandwidth is not utilized in the initial systems, eventual reductions must be kept in mind.

(d) The final system must be within the financial capabilities of all countries. For a variety of political and economic reasons, it is unrealistic to conclude that the U.S. could or should finance the satellite communication systems for other countries. This is tantamount to saying that the system should be of minimum cost, but it is actually a stronger statement. It implies that it is beneficial to wait a few years, if necessary, in order to develop a much more inexpensive system for the combined ground and satellite facilities. This particular point will affect the modulation system by perhaps forcing a design based upon a compromise between minimum power requirements and minimum bandwidth from (c).

From (a), (b), and (d), it would appear as though the stationary orbit would fulfill our requirements; however, random or phased orbits need not be ruled out at this time, even though the problems with a

*S. G. Lutz and S. Plotkin, NASA Report No. 4, "A Feasibility Study of Satellite Communication in the 15-20 Gc Frequency Range".

nonstationary system, besides being multitudinous, contain aspects for which solutions appear to be impossible. One of these problems is the interference while maintaining (b) above. Another is that the ground facility cost requirements are at least 10 times and perhaps 100 times greater than a similar facility for the stationary system.* One final point is that from (a), multiple-access capability for nonstationary orbit systems seems quite doubtful. However, like other engineering situations, a possible alternative cannot be ruled out because it contains problems.

III. MULTIPLEX SCHEME

Before discussing the modulation system, the multiplex system must be considered. Essentially, there are three known ways of multiplexing a number of separate channels: FDM, frequency division multiplex; TDM, time division multiplex; and orthogonal division or spread spectrum multiplex. Orthogonal division multiplex is used here in a broad sense and refers to any system in which a multitude of signals can be superimposed in frequency-time space. The receiver then utilizes some type of coherent detection which singles out one channel to the exclusion of all others. Systems which do this employ time-frequency address coding (see refs. 1 and 2), pseudo-random pulse coding (see ref. 3), and pseudo-random anologue carrier coding. In all of these there is fundamentally a band spreading mechanism which spreads each narrowband channel over a wideband in a noise-like fashion.

There are perhaps two major aspects to spread spectrum modulation in general. Because of the intermodulation aspects, the system can only be loaded to a fraction of its theoretical capacity. The reason for this is the threshold required for the receiver. A system which spreads the spectrum of a single channel by a factor of 1000, for example, could theoretically handle 1000 channels. However, if there is a 10 dB threshold in the receiver only 100 channels could be transmitted before seriously degrading the output. Thus, orthogonal modulation, in general, contains an inherent disadvantage. On the other hand, multiplexing is accomplished by coding, and since the number of codes available can be much larger than the theoretical loading capacity, it is possible to handle more channels if they are each lightly loaded. In the above example, if individual channels are only used 1 percent of

*It is intended that further work will be performed on this aspect of the problem.

the time, the system could then handle 10,000 channels and only 100 would be used on the average. During times when more than 100 are being used, the outputs would be noisy but still might be intelligible. Further, the 10,000 channels could be handled with random multiple access because each would have its own code. All receivers would have to be capable of detecting the entire 10,000 channels.

Contrasting the multiple-access capability of orthogonal modulation with FDM or TDM, it is noted that the latter two require a master control or its equivalent in order to allocate frequencies or time slots. Fundamentally, it would appear that the equipment complexity would thus be shifted from the individual receivers to the master control station. Whether or not there is more involved than this simple over-all analysis might be the subject of a future study and will not be treated any further here.

Jamming or interference protection is of prime importance in military systems but of only secondary consideration in commercial systems. In fact, as long as the output S/N ratio is satisfactory in the commercial system, it is undesirable to increase the bandwidth, transmitter power, or both, to obtain additional interference protection as would be the case in a military version. In view of this cursory analysis indicating that spread spectrum modulation will not provide any capability or advantages over the FDM or TDM systems in a commercial application, our attention will be confined to consideration of TDM and FDM only. Further consideration of orthogonal multiplexing as mentioned above may possibly be undertaken at a later date.

With regard to TDM, there are some formidable practical difficulties which have been recognized by the CCIR (see ref. 4). In fact, a direct quote seems applicable:

> "TDM has not, so far, been shown to be technically satisfactory for high-capacity radio-relay systems (300 telephone circuits or more)."

Of course, there is the possibility of using TDM with subgroups of channels, each less than 300, but then the effect would be to have many satellite systems instead of one with the loss of complete multiple access, one of our primary objectives. This same comment also applies to wideband FDM subgroup transmission. Fundamentally, the problem with TDM is to develop synchronization and switching techniques for nanosecond time intervals. It has been reported that work is being carried on in Great Britain in this regard at the present time, as well as limited development in the U.S.

It is noted that FDM is relatively straightforward, presents no serious problems for a multitude of channels, and is ideally suited for multiple access. Because TDM presents some serious developmental problems for which solutions do not appear to be imminent, the conclusion is to consider FDM only. However, should TDM become feasible and advantageous in the future, it can always be incorporated without substantially changing the power or bandwidth requirements.

Since we have decided to use FDM and are considering n telephone channels, the next question is whether the n channels are transmitted from a single ground station or a multitude. This affects the total ground transmitter power requirements as well as bringing to bear frequency overlap considerations. Actually, both situations must be allowed for from a multiple-access standpoint, and as we proceed, various possibilities will be mentioned with calculations where desirable. As for the satellite, all inputs are considered to be FDM which in turn are added together forming one composite signal for all n channels. These can then be retransmitted back as one super group. Ground receivers detect and filter out the channels they are interested in from the composite FDM wave.

A word regarding carrier spike interference is warranted at this point. It seems apparent that synchronizing or pilot frequencies will be required in the final configuration. These along with the carrier frequencies for the subchannels can be placed frequency-wise within the guard bands between ground microwave channels in order to minimize interference. This type of frequency allocation is quite convenient for FDM, but also may be applicable to TDM. The interference introduced by such carrier spikes is believed to be small enough so that it is not of major consequence so far as the modulation scheme is concerned. Interference from ground microwave systems should be much greater and solution of the frequency sharing problems could take care of the synchronizing frequency intrasystem interference as well.

IV. SYSTEMS TO BE CONSIDERED

Various papers treating this problem are refs. 5 through 8, which have all considered one or more of only three modulation systems: SSB, FM or PCM. Power and bandwidth requirements for other types of modulation systems are well known so that a detailed comparison here is not deemed warranted. Thus, this author agrees with the "boiling down" of the possibilities, and only wishes to add that ΔM, delta modulation, is beginning to be recognized and might very well offer significant advantages in the over-all picture as compared to digitally coded PCM. However, in this report only PCM is considered and in the final analysis, if pulse modulation appears advantageous, then a comparison with ΔM will be required. Of the three modulation systems, the comparison is made by calculating power and bandwidth requirements for each case. Figure 1 shows the general block diagram of the entire satellite system in which the master control, satellite relaying, when necessary, and frequency synchronizing are not shown. Also the encoding and decoding for PCM are contained in the modulator and demodulator respectively.

It is pointed out that PM and FM are so very similar that many papers do not distinguish between the two. In fact, a practical system will undoubtedly use emphasis and de-emphasis, which, for a basically FM system, the higher frequencies will be PM. Thus, FM will be used categorically here, but FM, PM, and mixtures of the two have been in mind.

V. COMPARISON OF INFORMATION EFFICIENCIES

The comparison of analogue and pulse systems from an information efficiency standpoint is not straightforward because of inherent fundamental differences. It is something like comparing "apples with oranges" to use a popular expression. However, a type of comparison can be made if one can obtain an equivalent bit rate for the analogue systems. For SSB, the approach is as follows for a single channel:

B_{ch} = information bandwidth

$1 + 2B_{ch} \approx 2B_{ch}$ = samples/sec required from the Sampling Theorem, ref. 9.

$\sqrt{\left(\frac{S}{N}\right)_{ch} + 1}$ = number of distinguishable voltage levels on the average, as seen from Fig. 2 where $\left(\frac{S}{N}\right)_{ch}$ is the ratio of average signal power to rms thermal noise power in a single channel; a 1 is added to account for the zero level, and voltage levels will be the square root of the distinguishable power levels. In an equivalent binary digital pulse system there would be k bits/sample, specifying 2^k levels; thus,

$$2^k \approx \sqrt{\left(\frac{S}{N}\right)_{ch} + 1} \qquad (1)$$

$$k \simeq \frac{1}{2}\log_2\left[1 + \left(\frac{S}{N}\right)_{ch}\right] \text{ bits/sample} \qquad (2)$$

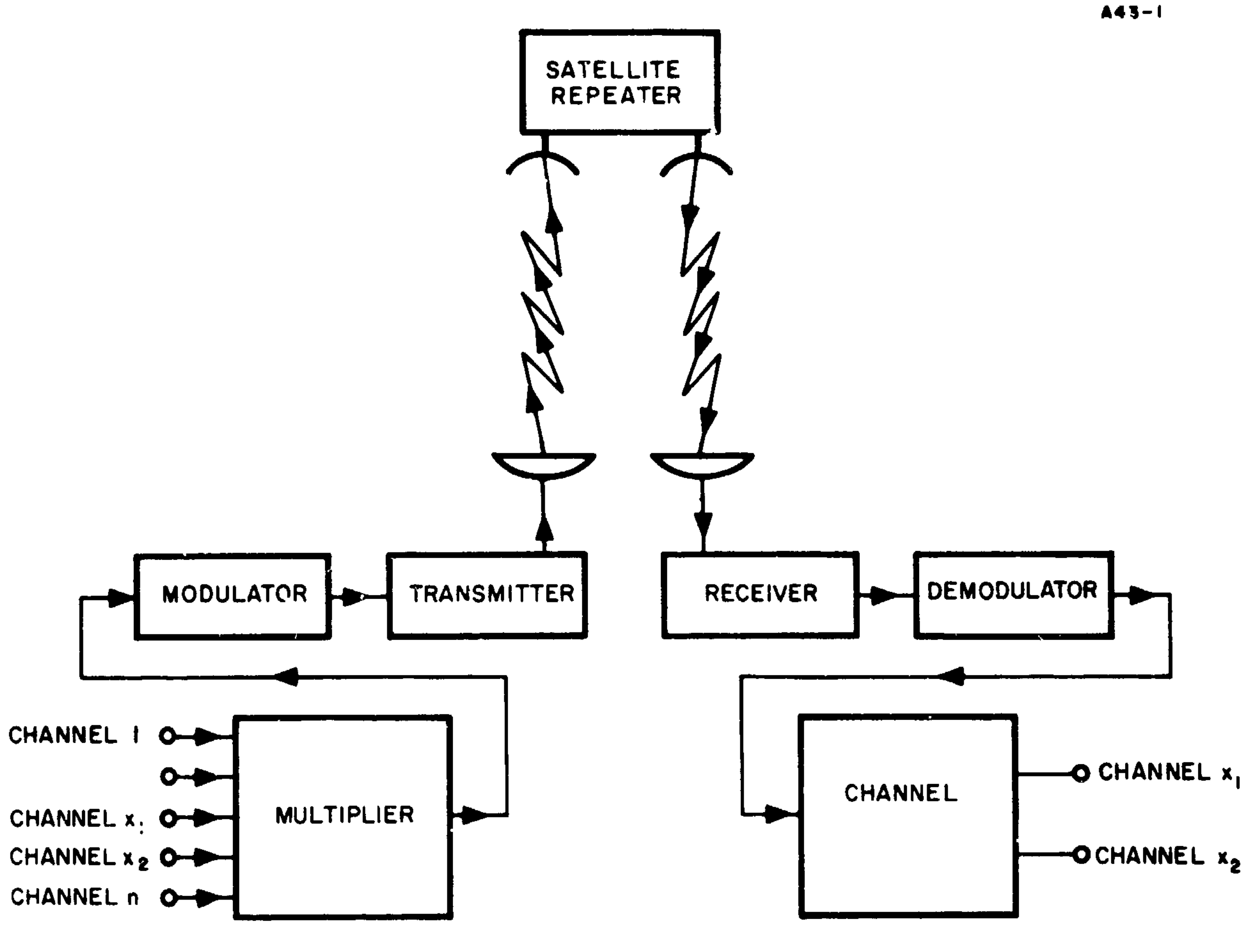

Fig. 1. Block diagram of a satellite communcation system.

$$C_{ch} = 2k\,B_{ch} \simeq B_{ch} \log_2 \left[1 + \left(\frac{S}{N}\right)_{ch}\right] \text{bits/sec} \qquad (3)$$

where C_{ch} is the theoretical minimum information rate required for one channel. Approximation signs are used because the expressions given are only valid on the average. For the heuristic argument presented here, one can now compare (3) with the ideal, theoretically maximum, rate (see ref. 9) using bandwidth occupancy as simply B_{ch}. One could then conclude that ideal SSB is 100 percent efficient from an information standpoint. In this report, the information efficiencies of the FM and PCM systems will be compared with ideal SSB and relative information efficiencies will be our concern. In an actual FDM system of n channels, an allowance for guard bands would be required and multiplexed SSB would thus be less than the ideal, but for a comparison of basic modulation system efficiency, this reduction can be neglected. Also for n channels FDM, the signal power is increased by the channel loading factor (see ref. 4) as well as the peak to rms loading (see ref. 10) and the thermal noise is increased by n. This gives a $(S/N)_c$ or composite peak signal-to-noise ratio and (4) results for the ideal SSB system.

$$C_{SSB} = n\,B_{ch} \log_2 \left[1 + \left(\frac{S}{N}\right)_c\right] \qquad (4)$$

which will be used for our comparison.

To obtain the information efficiency of FM, it is necessary to compare with SSB. In general, the FM will increase the bandwidth and decrease the transmitted S/N ratio as a function of the modulation index as given in Appendix A. Theoretically, FM requires infinite bandwidth but in practice a small amount of distortion is allowed and therefore a finite bandwidth is used. From ref. 9, the curve in Fig. 3 was derived which gives the bandwidth required for less than approximately 1 percent distortion.

Usual practice is to specify or limit the FM bandwidth to that given by (5) (see ref. 10*) which is much less than that given by Fig. 3. However, evaluation of the FM sideband components shows that B_{FM} in (5) deletes those sidebands from which contributions are less than about 4 percent instead of 1 percent. But since peak deviation and therefore peak bandwidth are only required a very small percentage of the

*Reference 10 uses $B_{FM} = 2f_m\,(\beta + 2)$; however, in most satellite communication work (5) is usually used because peak deviation and therefore peak bandwidth are only required a very small percentage of the time.

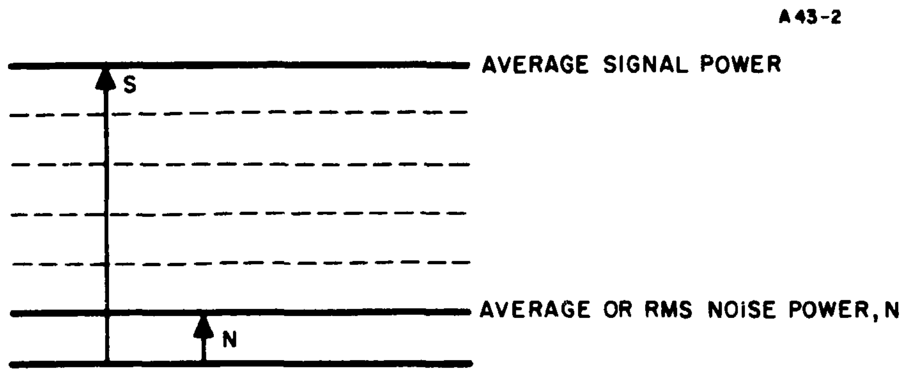

Fig. 2. Power level diagram.

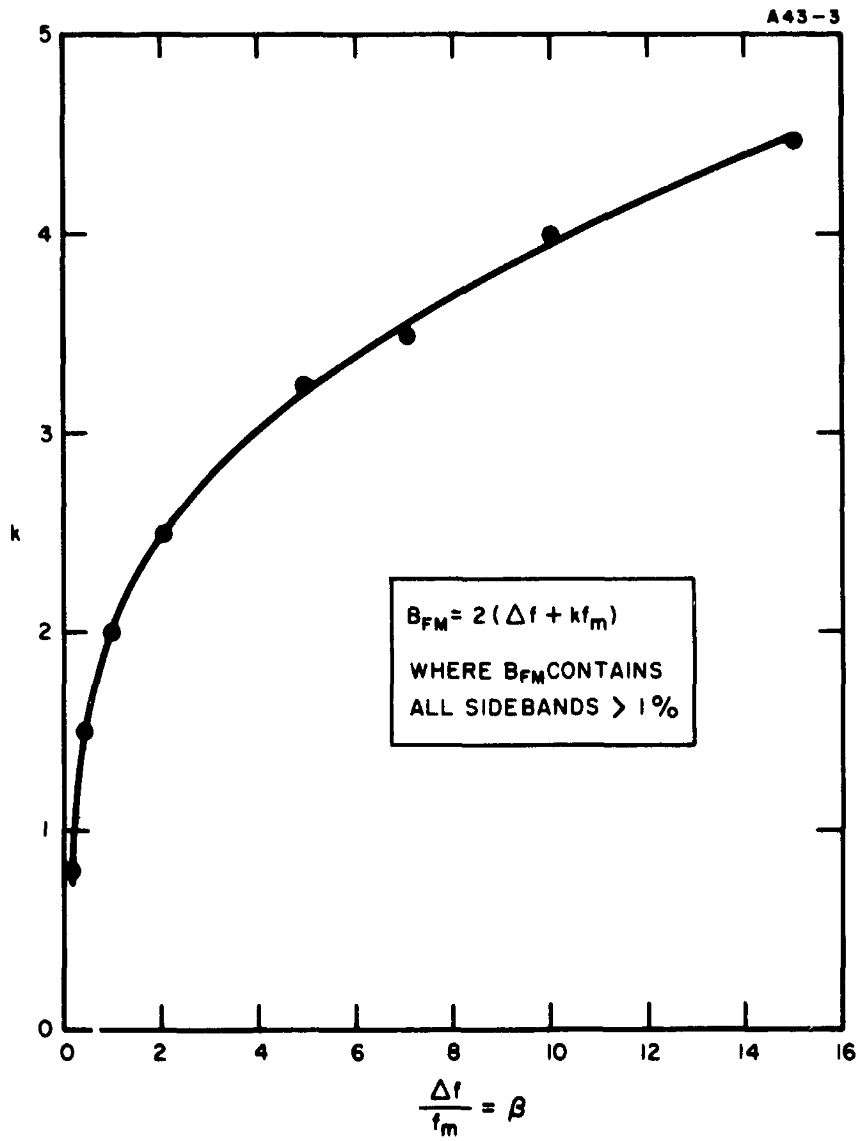

Fig. 3. FM bandwidth factor versus modulation index.

time, the FM bandwidth given by (5) will be considered acceptable in this report and the over-all distortion should be quite small, of the order of 1%.

$$B_{FM} \approx 2 \left[\Delta f + f_m\right] = 2 f_m (\beta + 1) \tag{5}$$

where Δf is the frequency deviation, f_m is the highest modulating frequency, and β is the modulation index. In the FDM situation, f_m will be $n B_{ch}$ and Δf can be absorbed in β. From (5) and (6A) of Appendix A we obtain (6).

$$\left(\frac{C}{N}\right) = \left(\frac{S}{N}\right)_c \times \frac{1}{3 \beta^2(\beta + 1)} \tag{6}$$

where (C/N) is the peak carrier-to-noise ratio, and peak power results when the loading factor, L_F, includes peak/rms loading as well as channel loading. The theoretical maximum information rate of the FM system is therefore:

$$C_{FM\ max} = 2 n B_{ch}(\beta + 1) \log_2 \left[1 + \left(\frac{S}{N}\right)_c \frac{1}{3\beta^2(\beta + 1)}\right] \tag{7}$$

and the FM information efficiency, Eff_{FM}, is $C_{SSB} / C_{FM\ max}$.

$$Eff_{FM} = \frac{\log_2 \left[1 + (S/N)_c\right]}{2(\beta + 1) \log_2 \left[1 + \frac{1}{3\beta^2(\beta + 1)} \left(\frac{S}{N}\right)_c\right]} \tag{8}$$

because the actual output information rate for both SSB and FM are the same.

For the FM feedback detection case, FMFB (see ref 11), the additional improvement as given in Appendix A (eq. 8A) reduces $(S/N)_c$. If the receiver bandwidth is three times the base band, (9) results.

$$Eff_{FMFB} = \frac{\log_2 \left[1 + \left(\frac{S}{N}\right)_c\right]}{2(\beta + 1) \log_2 \left[1 + \frac{1}{2\beta^2(\beta + 1)^2} \left(\frac{S}{N}\right)_c\right]} \tag{9}$$

Note that FMFB is only advantageous for large β.

In a PCM system we note that quantization noise is introduced by the modulation process and that thermal noise is introduced by the transmitting media and detection process. If the noise in the original signal is much less than the quantization noise and the transmitted (S/N) ratio is large enough so that errors in detection are negligible, the only noise in the output will be that due to quantization. This is treated in more detail later; suffice it to say that now we would like to show simply that thermal noise in the SSB system is effectively the same as quantization noise in PCM for the case of P_e, the error rate, being practically zero. This must follow because we assumed the number of distinguishable levels to be determined by the original thermal noise in the signal. From ref. 9 as well as other sources, the rms signal power to quantization noise power is given by (10).

$$\frac{S}{N_q} \text{ dB} = (1.76 + 6k) \text{ dB} \tag{10}$$

$$\frac{S}{N_q} = 3/2 \times 2^{2k} \tag{11}$$

$$\left(1 + \frac{S}{N_q}\right) = 2^{2k}\left[2^{-2k} + 1.5\right] \tag{12}$$

$$\log_2\left(1 + \frac{S}{N_q}\right) \simeq 2k + 0.585, \text{ for large } k \quad . \tag{13}$$

Since there are ideally 2k bits per second per cycle of baseband frequency,

$$\begin{aligned} 2knB_{ch} &= nB_{ch}\left[\log_2\left(1 + \frac{S}{N_q}\right) - 0.585\right], \text{ for large } k \\ &\simeq nB_{ch}\log_2\left(1 + \frac{S}{N_q}\right), \text{ for large } S/N_q \text{ and large } k \quad . \end{aligned} \tag{14}$$

In fact, had the actual rms power in the quantized sine wave signal been used rather than the rms of a sine wave, the answer would be exact rather than approximate. Since (14) is identical to (4), we have shown that quantization noise in PCM is identical to thermal noise in analogue systems, as anticipated, at least with respect to information.

PCM like FM is a modulation process which trades bandwidth for transmitted (S/N) ratio. Errors in detection are directly related to the transmitted (S/N) ratio and result in fluctuation or thermal-like noise in the output. Another source of noise is quantization noise which is solely related to the number of quantization levels or bits per sample in a binary code. This noise is present in the output regardless of the transmitted (S/N). In the end, both noise powers must be added up and the resulting output S/N is reduced. However, since a substantial reduction in output fluctuation noise (lower error rate, P_e) can be obtained by small increases in transmitted S/N, it seems appropriate to specify a transmitted S/N such that the only output noise of the system will essentially be due to quantization. It is therefore necessary to determine the transmitted S/N for a prescribed P_e In ref. 12 it is noted that for $P_e \approx 10^{-5}$, transmitted S/N of 10 dB is required. It is also noted parenthetically that there is little difference in performance between binary coherent detection and binary phase comparison detection as long as $P_e < 10^{-3}$, the difference in transmitted S/N being < 1 dB. Actually, from ref. 12, experimental data indicates a larger value of S/N than the theoretical calculations. Therefore, a value of 12 dB will be assumed here.

As a matter of completeness, it is pointed out that a different approach could be used. In ref. 13 a solution for output S/N as a function of transmitted S/N is given. This accounts for the fluctuation noise in the output caused by detection errors and neglects quantization noise. If we now decided to design the output S/N for fluctuation noise 10 dB higher than that for quantization noise, the latter would be the only essential noise in the output. Since we are using π phase modulation and ref. 13 considered only on-off pulses, a correction must be used. Several authors have included a factor of two for this correction; however, their rationale is not clear to this author. At any rate, including the factor of two and proceeding, it is noted that the required transmitted S/N is reasonably close to the 12 dB figure assumed above in the output S/N range of interest.

The next question is the bandwidth required. Ideally, it is shown in ref. 14 that the minimum bandwidth required would be the number of bits per sample times the baseband. In practice a more realistic value might be twice this value or perhaps 3/2. Using the value of twice minimum, the information efficiency of PCM, π phase modulation compared to SSB is given by

$$\mathrm{Eff}_{\pi\mathrm{PCM}} = \frac{\log_2 \left[1 + (S/N)_c\right]}{2k \log_2 \left[1 + 10^{1.2}\right]} \tag{15}$$

Since the $(S/N)_C$ for PCM cannot be continuously varying in this case, the (S/N_q) can be used in place of $(S/N)_C$. This substitution makes it possible to delete a correction term when using the following equations for comparing information efficiency of the three systems.

$$\mathrm{Eff}_{\mathrm{PCM}_\pi} = \frac{\log_2\left[1 + 10^{0.6k+0.2}\right]}{2k\log_2\left[1 + 10^{1.2}\right]} \tag{15a)*$$

which can be approximated for large k by

$$\mathrm{Eff}_{\pi\mathrm{PCM}} \approx 0.25 + \frac{1}{12k} \approx 0.25 \quad . \tag{16}$$

And for transmission with half the bandwidth by

$$\mathrm{Eff}_{\pi\mathrm{PCM}\ 1/2} \approx 0.50 + \frac{1}{6k} \approx 0.5 \quad . \tag{17}$$

Results using (8), (9), (16), and (17) are plotted in Fig. 4 which shows the information efficiency with respect to SSB. The results here show that for cases where wideband transmission is required because of transmitter power limitations, there is no clear cut choice between FM and PCM. Differences in information efficiency as used here eventually reflect themselves in the transmitter power requirements which are calculated later. That is, if the same bandwidth is used by both systems, the one with the highest information efficiency will yield the lowest transmitter power. It is also noted that the PCM efficiency is constant with S/N ratio which means that PCM trades bandwidth for (S/N) ratio exactly as the ideal capacity law stipulates. However, the FM systems can still be more efficient, power-wise, under certain conditions such as with smaller S/N ratios and larger than minimum bandwidth for PCM.

An example of how these curves might be used is as follows:

Let $(S/N)_c$ = 50 dB

k = 8 for the PCM system .

*1.76 was rounded off to 2.0.

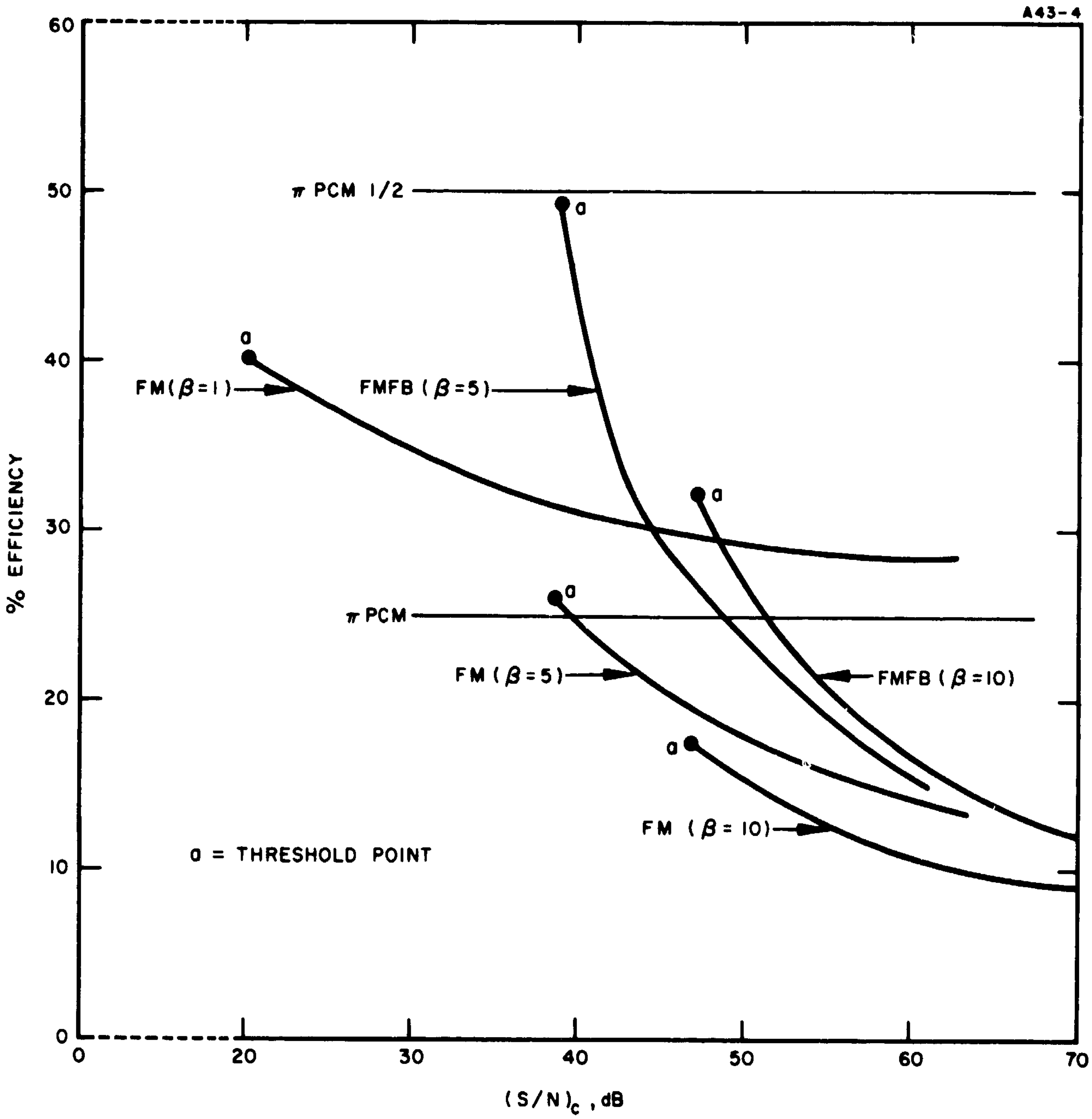

Fig. 4. Information efficiency relative to ideal SSB.

If π phase AM modulation is used, the relative bandwidth is $2k$. An FM system for equal bandwidth then has a modulation index of

$$\beta = \frac{2k}{2} - 1 = 7$$

Interpolating the curves of Fig. 4 for $(S/N) = 50$ dB and $\beta = 7$ gives

$$\frac{\mathrm{Eff}_{FM}\Big|_{\beta = 7}}{\mathrm{Eff}\Big|_{\pi PCM}} = \frac{0.165}{0.25} \approx \frac{\log_2\left[1 + (S/N)_{PCM}\right]}{\log_2\left[1 + (S/N)_{FM}\right]}$$

$$\left[1 + (S/N)_{FM}\right] = \left[1 + (S/N)_{PCM}\right]^{1.51}$$

$$(S/N)_{FM} \approx (S/N)_{PCM}^{\ 1.51}$$

and considerably more power would be required for a comparable (equal bandwidth) FM system (without feedback detection).

However, for the FM case where feedback detection is used and again equal bandwidth, the power requirements are approximately equal for πPCM($k = 8$) and FMFB($\beta = 7$).

Similar comparisons can be made with FMFB and PCM π phase modulation which is transmitted with ideal minimum or 3/2 minimum bandwidth. This latter has 1/2 to 3/4 the bandwidth of πPCM and therefore twice to 4/3 the efficiency. It is noted again that PCM is better than FMFB from an information efficiency standpoint for large $(S/N)_c$.

In concluding this section, it should be mentioned that information efficiency is not the only criterion for choosing a modulation system. Such practical considerations as nonlinear amplifier effects, switching and synchronizing problems, feasibility of wideband feedback detectors, and effects of component inaccuracies have not been considered. However, given a set of specifications as in the above example, one can calculate the relative power magnitudes.

VI. TRANSMITTER POWER REQUIREMENTS

Allowable noise power for satellite communication has not as yet been decided; however, for a 2500 km ground radio relay link, a value of 10,000 pW psophometrically weighted total noise power (averaged over an hour with 0 dBm reference) has been specified. If the same should apply to satellite communications and only a small fraction allowed for the satellite link (assuming ground relaying to the satellite earth stations), a reasonable value can be obtained. Therefore, let the satellite link have 1000 pW of noise or 10% of the total allowable for a signal power of 1 mW both at the point of zero reference. This results in a (S/N) ratio psophometrically weighted of 60 dB, or 57.5 dB unweighted.* One could now proceed to calculate the power required for each case but it will turn out that an inordinate amount of power will be required for SSB. In any practical situation the use of compandors to improve the S/N ratio will be used and it is noted that these may be used to advantage with all three modulation systems. Since a companding improvement of 13.5 dB is not unreasonable, it will be assumed that a desired 44 dB S/N ratio per channel is required from the modulation or transmission system, i.e., $(S/N)_{ch} = 44$ dB at the receiver output before expanding or de-companding.

Given a number of channels to be transmitted, n; these can be sent as one group or a number of subgroups, $p = n/m$, where m is the number of channels in each subgroup. Because of the multiplexing and variations of power with bandwidth, there is not a linear relation between the power in n channels and the power in m channels. For our purposes we will sometimes consider simply $p = 1$ and calculate the power required for arbitrary n channels without losing generality. The total power required will be simply p times the power for the m channel subgroups and here we will simply use n to indicate the total number of channels in the particular group to be transmitted.

Assumptions used in the following calculations are as follows:

$T°$ = receiver noise temperature = 300°K

f = transmitting frequency = 6 Gc

D = distance from satellite to earth at the edge of the coverage region; receiver antenna raised a minimum of 5° and the orbit height is 22,240 miles = 25,600 miles

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*Or 56.4 dB unweighted for each voice channel being restricted to 3.1 kc out of the 4 kc alloted.

G_s = satellite antenna gain when the 3 dB point is at the edge of the coverage region = 19.5 dB at beam center

G_R = earth antenna gain for 40 ft aperture = 55 dB

L = miscellaneous losses
= 3 dB, polarization
+3 dB, antenna gain reduction at coverage edge
+4 dB, coupling, mixers, atmospheric absorption, etc.
= 10 dB total

B_{ch} = bandwidth per channel = 4 kc

$(S/N)_{ch}$ = 44 dB

When n channels are multiplexed FDM, the signal power is increased which is termed the channel loading factor, (18), and specified by the CCIR in ref. 4. In addition, the peak/rms ratio changes as n varies according to curves given in ref. 10 which in turn were derived from data in ref. 16. This is an overload factor and must be specified for the peak value being exceeded for x percent of the time. A simple derivation in Appendix B shows that a large number of speech channels FDM truly approach white noise. The formulas in (19) were obtained by simple curve fitting to those in ref. 10 and reproduced here as Fig. 5.

$$\text{Channel loading} = (-15 + 10 \log_{10} n) \text{ dB}, \; n > 240$$

$$= (-1 + 4 \log_{10} n) \text{ dB}, \; 12 < n < 240 \qquad (18)$$

$$\text{Peak/rms loading} = (10.2 + 9_e^{-0.093\,n}) \text{ dB}, \text{ for } x = 0.1\%$$

$$= (8.3 + 4.7_e^{-0.1\,n}) \text{ dB}, \text{ for } x = 1.0\% \qquad (19)$$

VII. SSB CALCULATIONS

P_T = peak transmitter power in dB

$$= A + L - G_R - G_s + (S/N)_t + P_N \qquad (20)$$

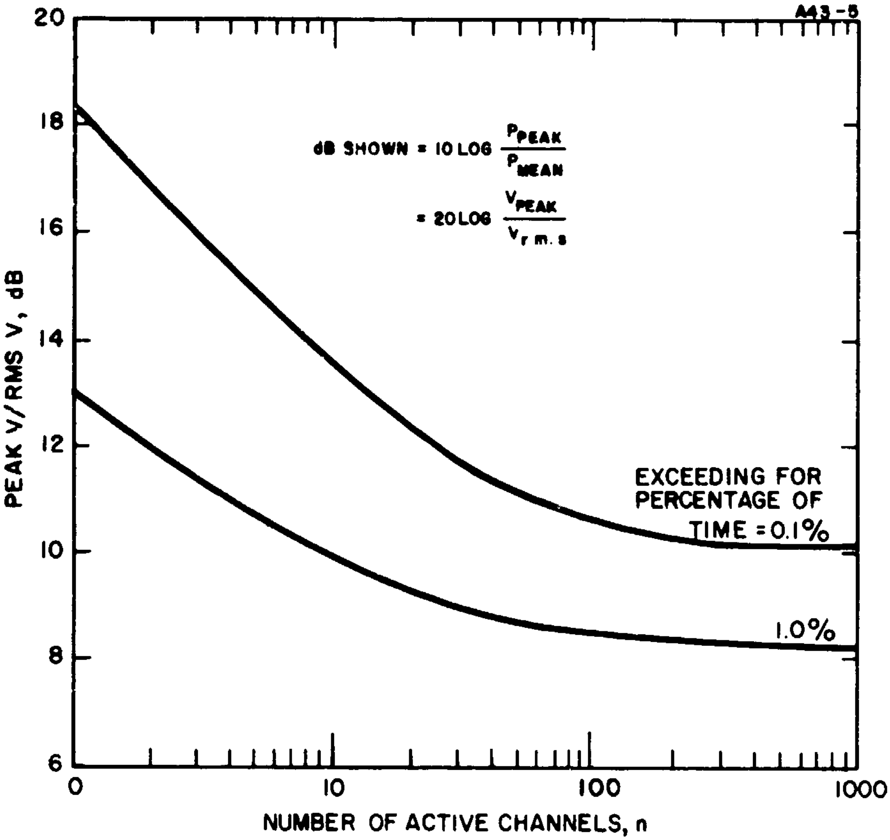

Fig. 5. Peak to RMS loading factor. (These curves are for average amplifiers in Holbrook and Dixon tests; see Ref. 16.)

where

$$A = \text{path losses}$$

$(S/N)_t$ = transmitted (S/N) ratio or carrier to noise ratio

P_N = total noise power at the receiver output but referred to receiver input

$$A = 37 + 20 \log_{10} f_{Mc} + 20 \log_{10} D_{miles}$$

$$= 37 + 60 + 15.5 + 80 + 8.2 = 200.7 \text{ dB} \quad (21)$$

From (18), (19), and increasing the noise bandwidth n times,

$$(S/N)_t = (S/N)_{ch} + L_F - 10 \log_{10} n \quad (22)$$

where L_F is the total load factor

$$P_N = 10 \log (KTnB_{ch}) = 10 \log (1.38 \times 10^{-23} \times 300 \times 4,000 \times n)$$

$$= 10 \log_{10} n + (-170) + 2.2 = (-167.8 + 10 \log_{10} n) \text{ dBW} \quad (23)$$

Thus P_T is reduced to being a function of n only.

$$P_{T_{SSB}} = 200.7 + 10 - 55 - 19.5 + \left[(S/N)_t\right] - 167.8 + 10 \log_{10} n \quad (24)$$

$$= 31.6 + \Big[(S/N)_{ch} + (-15 + 10 \log_{10} n)$$
$$+ 10.2 + 9\, e^{-0.093\, n} - 10 \log n\Big]$$
$$+ 10 \log_{10} n, \text{ for } n > 240 \text{ and } 0.1\% \text{ overload} \quad (25)$$

$$= -31.6 + \Big[(S/N)_{ch} + (-1 + 4 \log_{10} n) + 10.2$$
$$+ 9\, e^{-0.093\, n} - 10 \log n\Big]$$
$$+ 10 \log_{10} n, \text{ for } 12 < n < 280 \text{ and } 0.1\% \text{ overload.} \quad (26)$$

Combining the $\log_{10} n$ terms and inserting the value of $(S/N)_{ch}$ yields

$$P_{T_{SSB}} = 7.6 + 9\, e^{-0.093n} + 10 \log_{10} n$$

$$= 7.6 + 10 \log_{10} n, \text{ for } n > 240 \text{ and } 0.1\% \text{ overload} \quad (27)$$

$$= 21\ 6 + 9\, e^{-0.093n} + 4 \log_{10} n,$$

$$\text{for } 12 < n < 240 \text{ and } 0.1\% \text{ overload.} \quad (28)$$

For $n = 1$ there is no channel loading and 18.4 dB peak/rms loading, therefore

$$P_{T_{SSB}} = -31.6 + 44 + 18.4 = 30.8 \text{ dBW, for } n = 1 \ . \quad (29)$$

One curve based upon (27), (28), and (29) is shown in Fig. 6, curve (a), for the SSB case. The same approach was used for the case of overload 1% of the time and these results are given in Fig. 7.

VIII. FM CALCULATIONS

Because of the enhanced frequency band required for FM, there are several ways p subgroups can be treated.

(a) Each subgroup can transmit its m FDM channels FM about a different carrier. The total input to the satellite receiver is then p FDM subgroups, each of which is FM. The satellite can then detect the m channels of each subgroup, FDM the entire n channels, and retransmit the n channels FM as one supergroup. Ground reception is accomplished by one detection of the composite FM followed by one narrowband filter for each channel output.

(b) Same as (a) except that the satellite merely retransmits its input. The earth stations are all required to perform the additional signal processing which in (a) is only performed once in the satellite. Satellite transmitting power and bandwidth would be considerably increased in this case, but the satellite processing equipment is deleted.

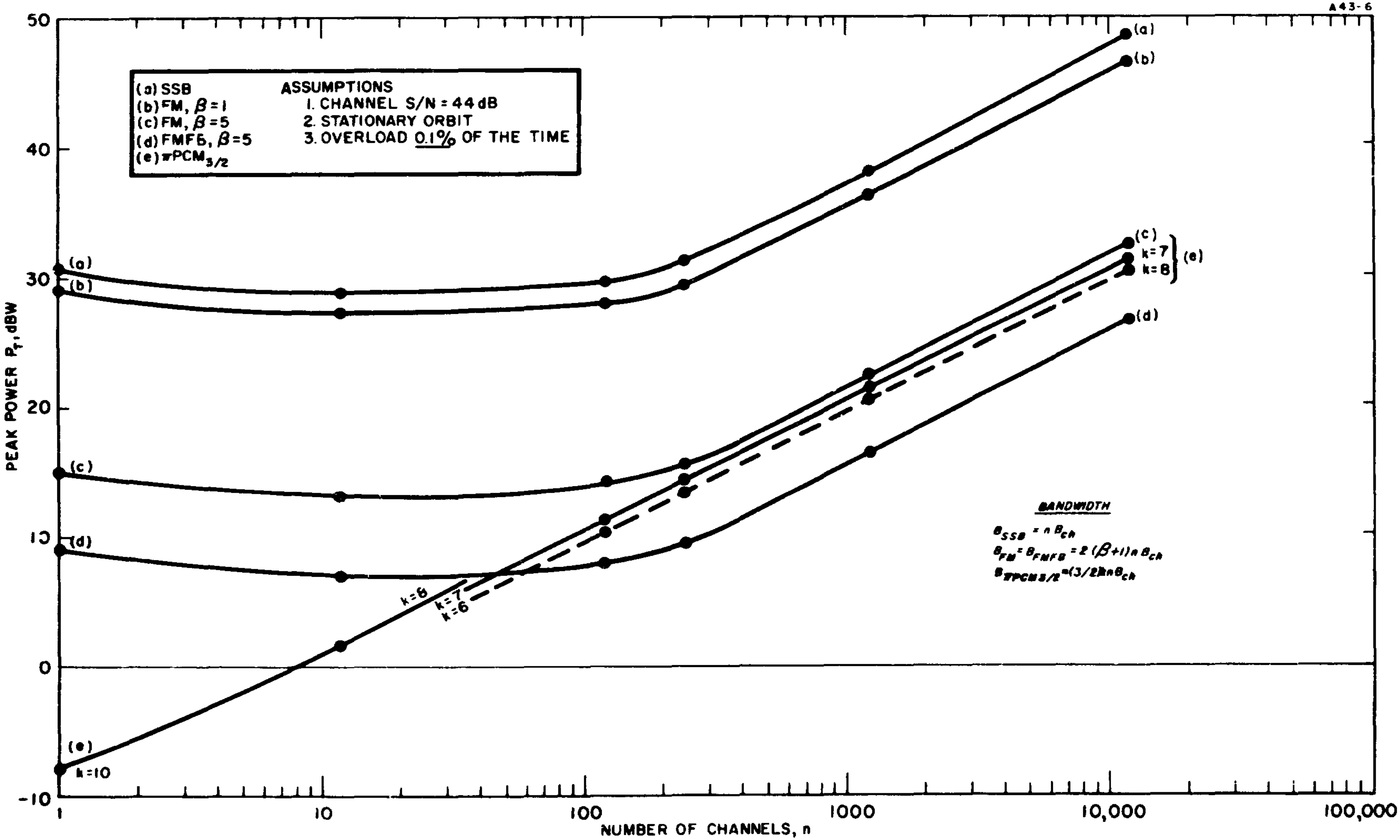

Fig. 6. Peak transmitter power versus number of channels.

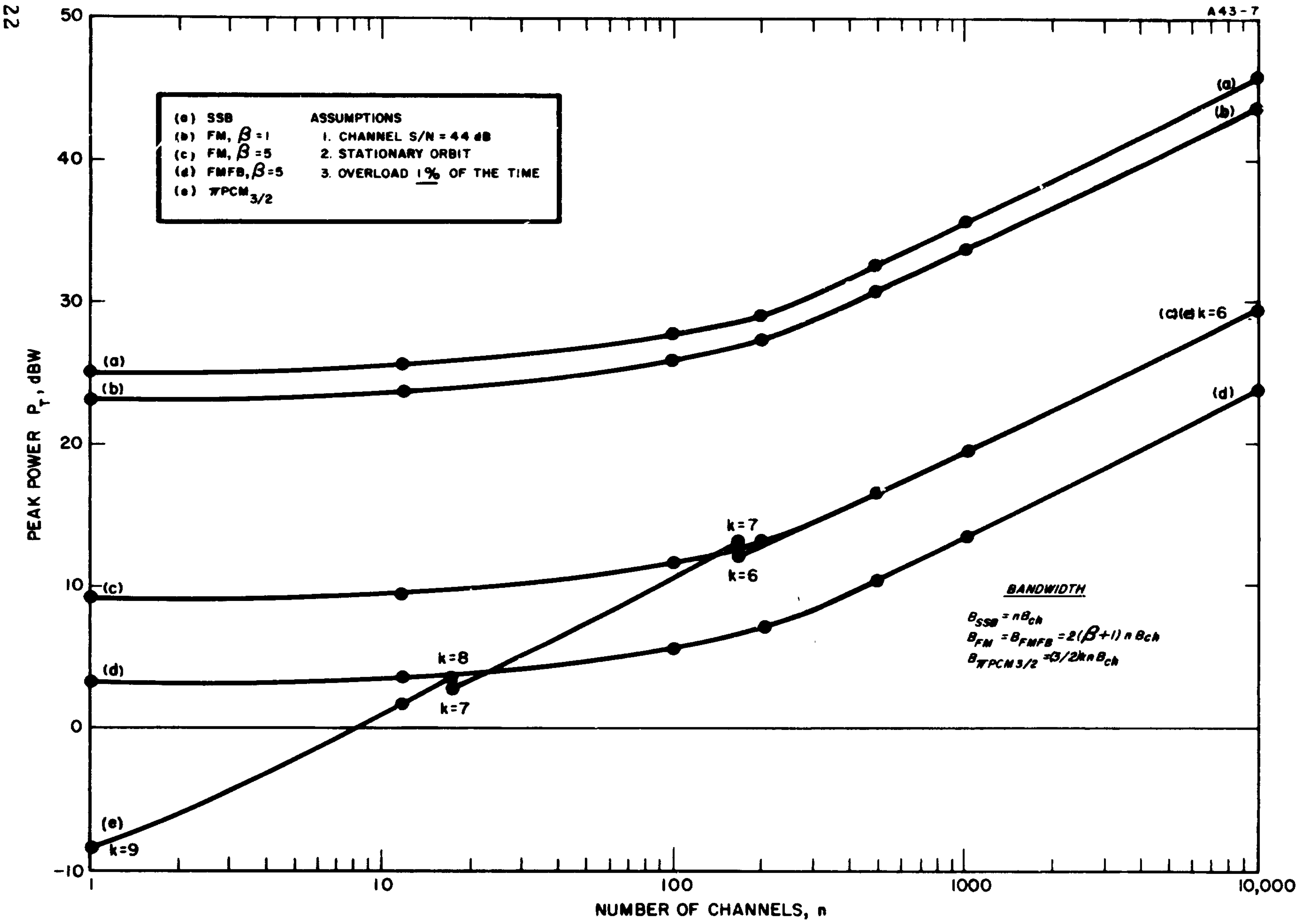

Fig. 7. Peak transmitter power versus number of channels.

(c) Similar to (b) except that the FM subgroups themselves are FDM and the same carrier is used for FM transmission of each subgroup. The satellite merely retransmits its input and the earth reception is the same as that of (a). Inordinate amounts of power are required here for the original earth transmitter of the highest frequency subgroup. The fundamental reason for this is the triangular shape of the noise spectrum which for the top frequency FM subgroup would provide a degradation in S/N ratio rather than an improvement.

Comparison of these three possibilities for FM transmission of a multitude of independent subgroups leads to the conclusion that (c) is not advantageous and a decision regarding the other two is a moot point at present. However, assuming that minimum satellite transmitter power because of interference factors is the prime consideration, the situation as outlined in (a) would be best. Results here can be used for either of the two methods, the only difference being in the transmitter power for the down link.

Calculation of transmitter power requirements are obtained similarly to those for SSB, except that FM improvement factors must be included as shown in Appendix A. The final result is simply

$$P_T = (S/N)_{ch} + L_F - 10 \log n - 20 \log_{10} \sqrt{3/2}\ \beta - 10 \log_{10} 2(\beta + 1) + A + L - G_R - G_S + P_N \tag{30}$$

where

$$\begin{aligned} P_N &= 10 \log_{10}(kTB_{FM}) = 10 \log_{10} 1.38 \times 10^{-23} \times 300 \times 4000 \\ &\quad + 10 \log 2(\beta + 1) + 10 \log n \\ &= \left[-167.8 + 10 \log 2(\beta + 1) + 10 \log n\right] \text{ dBW} \end{aligned}$$

and a receiver temperature of 300°K has been assumed as before. For different temperatures, one merely scales the final answer. If we include the previous values for A, G_R, G_S, $(S/N)_{ch}$, and L_F,

$$\begin{aligned} P_{T_{FM}} &= (19.8 + 4 \log n + 9\, e^{-0.093n} - 20 \log \beta) \text{ dBW}, \quad 12 < n < 240 \\ &= (5.8 + 10 \log n - 20 \log \beta) \text{ dBW}, \quad n > 240 \quad . \end{aligned} \tag{31}$$

For one channel, there will be no channel loading but 18.4 dB peak/rms loading which results in

$$P_{T_{FM}} = (29 - 20 \log_{10} \beta) \text{ dBW, for } n = 1 \, . \tag{32}$$

For feedback detection, the receiver noise power is limited to the noise in approximately three times the baseband instead of the full FM bandwidth. This simply adds a correction term.

$$P_{T_{FMFB}} = P_{T_{FM}} + 10 \log 3nB_{ch} - 10 \log 2nB_{ch}(\beta + 1)$$

$$= P_{T_{FM}} + 1.8 - 10 \log(\beta + 1) \quad . \tag{33}$$

According to (31), (32), and (33), one could increase β indefinitely and reduce the $P_{T_{FM}}$ arbitrarily while maintaining a constant $(S/N)_{ch}$. This, of course, is not true because of the FM threshold. As the C/N drops below approximately 12 dB, one ceases to realize the FM improvement factor. Thus, for the above example, there is a maximum value of β given by (34) for $n > 240$.

$$(S/N)_{ch} + L_F - 10 \log n - 10 \log 3\beta^2(\beta + 1) > 12 \text{ dB} \tag{34}$$

where

$$(S/N)_{ch} - 15 + 10.2 - 10 \log 3\beta^2_{max}(\beta_{max} + 1) \simeq 12 \text{ dB} \tag{35}$$

and for a 44 dB channel signal to noise ratio

$$\beta_{max} \simeq 5.6 \quad . \tag{36}$$

Therefore a $\beta = 5$ will be used as a maximum value, keeping in mind that this can be increased only if the $(S/N)_{ch}$ is increased.

FM transmitter power curves based on (31), (32), and (33) are given in Fig. 6. Similar results for overload 1% of the time is given in Fig. 7. An example assuming method (a) would be as follows using Fig. 6:

$$n = 1200$$
$$p = 100$$
$$m = 12 \quad .$$

For the up link, letting $\beta = 1$, the $P_{T_{up}} = 100 \times 29$ dBW or 80 kW total, because P_T for $m = 12$ is 29 dBW. (If method (b) were used, several additional dB would be required because the satellite doesn't remodulate the signal and the noise is additive).

Assuming an earth receiver noise temperature of 30°K for the down link (rather than the 300°K for the up link) results in 10 dB less power. However, if β is increased to 5 in the satellite remodulation process and all 1200 channels are transmitted as one composite signal,

$P_{T_{FM}\,\text{down}}$ = (22. 6 dBW - 10 dB) = 12. 6 dBW or 18. 2 W and

$P_{T_{FMFB}\,\text{down}}$ = (16. 6 - 10) dBW or 4. 6 W. These numbers are for overload 0. 1% of the time (Fig. 6).

In concluding this section on FM, it is noted that peak power is being transmitted at all times and that peak power is directly related to the peak signal power required. At peak deviation nearly all power is signal or sideband power whereas for small deviation or small signal power considerable carrier power is wasted. Another point worth mentioning is that FM threshold is reached very gradually so that intelligible communication can still be received many times even though the operation is considerably below threshold. This is contrasted with the threshold of PCM which is quite abrupt.

IX. PCM CALCULATIONS

Because PCM is essentially wideband transmission similar to FM, the same three possibilities for transmission of subgroups are applicable. However, the results for method (c) are not the same because PCM noise is flat rather than triangular. Thus method (c) would not suffer from the same disadvantage as in FM; the same transmitter power is required as for the other methods, no satellite processing is required, and the earth receiver can detect the composite n channels at once. The down link power is merely scaled by the receiver noise temperatures. In practice the ground transmitter power should be increased by several dB in all modulation systems where the satellite does not remodulate the composite wave (methods (b) and (c)). Noise is additive so that a higher carrier to noise is required at the satellite input to achieve the desired value at the earth receiver. As in the example for FM transmission, transmitted power in the down link is merely scaled by the receiver noise temperatures when no signal processing (remodulation) is performed in the satellite.

For PCM here, π phase modulation is assumed which requires 3/2 ideal minimum bandwidth. This results in approximately 4/3 the information efficiency as calculated for Fig. 4.

$$P_T = (S/N)_{\pi PCM\ 3/2} + A - G_R - G_s + L + P_N \tag{37}$$

$$B_{\pi PCM\ 3/2} = 3/2\ nkB_{ch} \tag{38}$$

$$(S/N)_{\pi PCM\ 3/2} = 12\ dB \tag{39}$$

$$P_N = \left(-167.8 + 10 \log \frac{3n\,k}{2}\right) dB \tag{40}$$

Note that P_N changes with the noise bandwidth which is the system bandwidth for all cases except FMFB. Using transmission method (a) given previously and the same specifications in (37) as used before, the following results are obtained for πPCM 3/2.

$$(S/N)_{PCM} = (S/N)_{ch}\ dB + L_F - 10 \log n \approx (6k + 2) \tag{41}$$

$$P_N = -167.8 + 1.8 + 10 \log k + 10 \log n = (-166 + 10 \log nk)\ dBW \tag{42}$$

$$A + L - G_s - G_R = 200.7 + 10 - 19.5 - 55 = 136.2\ dB\ . \tag{43}$$

The quantity k must be found from (41) and is the next highest integer from that obtained when using the equality sign. For overload 0.1% of the time,

$$\begin{aligned}(S/N)_{PCM} &= 44 + (-15 + 10 \log n) + 10.2 \\ &\quad + 9\, e^{-0.093n} - 10 \log n \text{ for } n > 240 \\ &= 44 - 1 - 6 \log n + 10.2 + 9\, e^{-0.093\, n}, \\ &\quad \text{for } 12 < n < 240\end{aligned}$$

or for the actual system, from (41), solving for k gives

$$\begin{aligned}k &\approx 6.2 + 1.5\, e^{-0.093n} \simeq 7,\ n > 240 \\ &= 8.53 - \log\ n + 1.5\, e^{-0.093n},\ 12 < n < 240\end{aligned}$$

$$\text{so that } k = 8,\ 12 < n < 35$$
$$= 7,\ n > 35$$

$$\text{and } (S/N)_c = 50 \text{ dB},\ 12 < n < 35$$
$$= 44 \text{ dB},\ n > 35$$

The actual output of the PCM system will have somewhat better S/N ratio than that specified because of the smaller quantization noise here than thermal noise in the analogue systems. In fact, one could use $k = 6$ for $n > 60$ with the result that output $(S/N)_{ch}$ would be only slightly less than 44 dB for the PCM system.

From the above values of k, (42), (43), and 12 dB for (C/N) ratio,

$$P_{T_{\pi PCM\ 3/2}} = (12 + 136.2 - 166 + 10 \log n + 10 \log k) \text{ dBW} \tag{44}$$

$$= (-17.8 + 10 \log n + 10 \log k) \text{ dBW}$$

$$= (-8.8 + 10 \log n) \text{ dBW},\ 12 < n < 35 \tag{45}$$

$$= (-9.3 + 10 \log n) \text{ dBW},\ n > 35,\ (S/N)_{ch} > 44 \text{ dB} \tag{46}$$

$$= (-10 + 10 \log n) \text{ dBW},\ n > 60,\ (S/N)_{ch} \lesssim 44 \text{ dB} \tag{47}$$

For one channel, as before, there is no channel loading but 18.4 dB peak/rms loading. Solving for k in (41) gives $k \approx 10$. Therefore,

$$P_{T_{\pi PCM\ 3/2}} = -7.8 \text{ dBW for one channel} \ . \tag{48}$$

A curve of (45), (46), (47), and (48) is shown in Fig. 6. An identical approach for overload 1% of the time is shown in Fig. 7.

X. COMPARISON

It is noted in Fig. 6 that PCM requires more power than $FMFB|_{\beta = 5}$ and n > 50 or 60. Fundamentally, the reason for this is that the bandwidth for FM is much greater; in fact, a comparison of bandwidths is given in Table I.

TABLE I

Relative System Bandwidths

System	Bandwidth
SSB	1
FM	$2(\beta + 1)$
πPCM 3/2	1.5 k

If FM and PCM are compared for the same system bandwidth, the $\beta = 4.25$ when k = 7. Letting n = 1200, $(S/N)_{ch}$ = 44 dB, or $(S/N)_c$ = 39.2 dB for overload 0.1% of the time,

$$P_{T_{FM}} = 24 \text{ dBW}$$

$$P_{T_{FMFB}} = 18.6 \text{ dBW}$$

$$P_{T_{\pi PCM\ 3/2}} = 21.5 \text{ dBW}$$

For larger (S/N) ratios the PCM system eventually requires less power than FMFB. This follows from Fig. 4 when the PCM curve is increased by 4/3 for comparison.

The use of Fig. 4 is straightforward. However, it is noted that had (15) been used for the PCM efficiency curves, a comparison of efficiencies would only be exact at the $(S/N)_c$ values of 38 dB, 44 dB, 50dB, etc. In that case an additional correction term of

$$\frac{(S/N)_c \text{ dB}}{6k + 2}$$

would have to be included in order to calculate the difference in peak power requirements directly from the information efficiency curves. Since (16) and (17) were used for the curves, no correction term is required. For the previous equal bandwidth example, the PCM system ($k = 7$) yields a 4.8 dB larger $(S/N)_{ch}$, (44-39.2), than the FMFB system ($\beta = 4.25$).

$$P_{T_{\pi PCM\ 3/2}} = \left(P_{T_{FMFB}}\right)^{y}$$

where $$y = \frac{Eff_{FMFB}}{Eff_{\pi PCM} \times 4/3} = \frac{0.385}{0.25 \times 4/3} = 1.15 \quad .$$

Another possibility of course would be to allow a slightly smaller (1.2 dB), $(S/N)_{ch}$ for the PCM system by letting $k = 6$. The FM modulation index, β, would then be 3.5.

TABLE II

Comparison of $\pi PCM_{3/2}$ with FMFB for Equal Bandwidths

S/N_c	38	44	50	56	62
k	6	7	8	9	10
β	4.5	5.25	6	6.75	7.5
$Eff_{\pi PCM\ 3/2}$	0.33	0.33	0.33	0.33	0.33
Eff_{FMFB}	0.437	0.32	0.245	0.194	0.159

Using the values in Table I, (9) and (16) after multiplying by 4/3, the results in Table II are obtained. These results are plotted in Fig. 8 and show that for $(S/N)_c$ greater than 44 dB in a "practical" system, PCM requires less peak power than FMFB; and for $(S/N)_c$ less than 44 dB, the reverse is true. As one changes various factors considered to be practical, the break point between PCM and FMFB changes somewhat. This break point does not change very much, however, because of the exponential nature of information efficiency relative to signal power requirements. The "practical" aspects assumed here were that PCM would require 3/2 its minimum theoretical transmission bandwidth and that FMFB requires 3/2 its minimum theoretical detector intermediate frequency bandwidth. Also the transmitted (S/N) for PCM was taken as 12 dB where the theoretical value might be 9 or 10 dB.

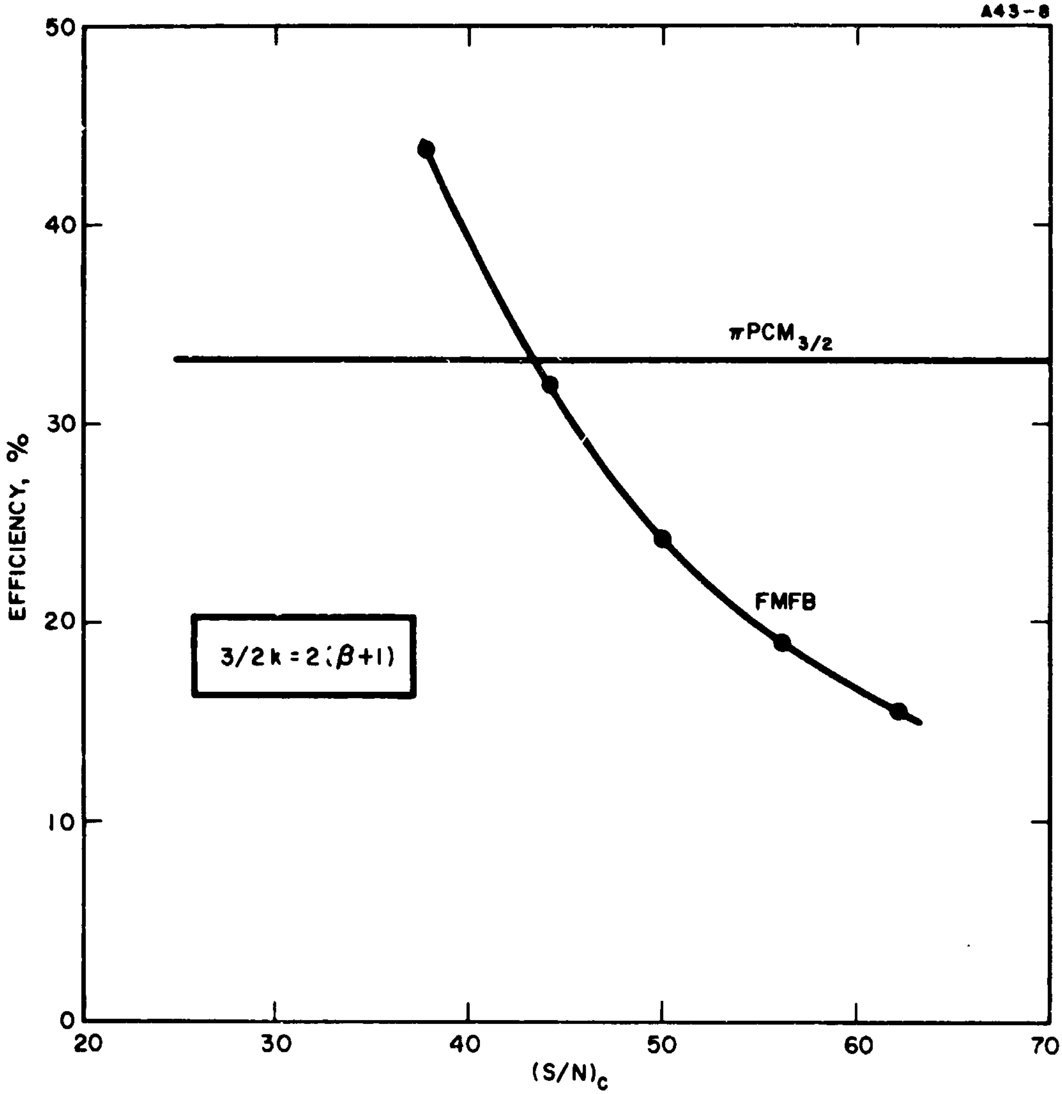

Fig. 8. Practical PCM and FMFB efficiencies for equal bandwidth systems.

XI. APPLICATION TO TV

One usual specification is that the satellite communication system must be able to transmit one TV channel instead of 1200 telephone channels. Reference 4 gives the CCIR recommendation for 625 line, 5 Mc, TV systems of 52 dB weighted $(S/N)_{TV}$ ratio where

$$(S/N)_{TV} = \frac{\text{peak-to-peak video signal power}}{\text{rms noise power in total frequency band}} \quad . \tag{49}$$

Information about the weighting network is given in CCIR recommendation No. 267 where the dB weighting values of 8.5 dB for "white" noise and 16.3 dB for "triangular" noise is given. Also (49) considers peak-to-peak signal power because the TV video signal is of one polarity as compared to speech which is composed of double polarity waves. However, in previous power calculations for the speech channels, a 10.2 dB average peak-to-rms factor was included. Therefore, it appears appropriate to compare a 1200 channel speech system calculated previously with the TV system specifications directly without changing the signal power. Thus a

$$\begin{aligned} (S/N)_{TV} &= 52 - 8.5 = 43.5 \text{ dB unweighted for flat noise} \\ &= 52 - 16.3 = 35.7 \text{ dB unweighted for triangular noise} \end{aligned} \tag{50}$$

is required, whereas previous calculations were for a S/N ratio in the composite 1200 speech channel system of 39.2 dB. The result then is an increase of 4.5 dB in transmitted power in SSB transmission in order to accommodate one TV channel within present CCIR specifications for ground microwave relay systems. FM power requirements are 3.5 dB higher than required because the noise is triangular; PCM, on the other hand, requires only about 1 dB increase in power.

It is pointed out that CCIR recommendations for tropospheric scatter TV transmission have not been made as yet. According to ref. 4, a smaller value of $(S/N)_{TV}$ is anticipated because all CCIR recommendations represent compromises between quality and cost. In the light of severe increases in cost, lower quality performance is generally considered acceptable.

Another point is that recommendations for a 4 Mc, 525 line system have not been made whereas the recommended $(S/N)_{TV}$ for a 3 Mc, 405 line system is only 50 dB weighted. Thus, one might expect that a smaller value of $(S/N)_{TV}$ would be acceptable for a TV system based on present USA standards.

Recommendations sponsored by the USA argue that the above CCIR requirements are too low. The U.S. proposal is to increase the $(S/N)_{TV}$ unweighted to 48 dB for "flat" noise and 43 dB for "triangular" noise. This, of course, would increase the power requirements of SSB appreciably as well as having smaller but increasing effects on FM and PCM power transmitted.

XII. MISCELLANEOUS ASPECTS

Several factors tending to increase the output S/N ratio have been omitted or treated conservatively. One of these is the use of pre-emphasis which would provide a 4.8 dB increase in S/N. Another factor is the earth station antenna gain of 55 dB which means approximately a 40-ft diameter parabolic dish at 6 Gc. This is indeed quite conservative since 60-ft diameter dishes are quite practical yielding 58.6 dB gain. In fact, even larger diameters are quite feasible, although from a practical standpoint, antenna gains are limited to about 60 dB at present. Thus the calculations would perhaps appear to be too conservative and a reduction in power by as much as 10 dB could have been argued. Further mention should be made that the 19.5 dB satellite antenna gain assumes complete coverage of the earth within view. If channels are beamed to smaller areas, then more highly directional satellite antennas can be used resulting in additional system gain. Another further aspect is that special satellite antennas might be developed which would eliminate the 3 dB gain reduction at the edge of the coverage area.

It is pointed out that the calculations were based upon T° = 300°K. If the satellite receiver noise temperature is 3000°K, then 10 dB must be added to the ground transmitter power. On the other hand, if an earth receiver noise temperature were 60°K, a reduction of 7 dB can be taken from the satellite transmitter power. As mentioned before, if the active satellite repeater in non-regenerative, approximately 3 dB must be added to both of the transmitter powers in order to maintain noise power below the relative 1000 pW assumed in the first place. Still another 3 dB increase in transmitter power might have been included for the peak/rms loading factor because the average peak loading was assumed instead of the largest expected value. On the other hand, peak loading was considered for no overload 99.9% of the time in all of the examples, whereas for no overload 99% of the time, approximately 2 dB less peak power loading results. Fading margins were also not included which would result in a further increase in transmitter power.

A baseband of 4.8 Mc for the 1200 channels implies that the lowest band approaches zero frequency. To avoid the transmission of dc, a baseband of 5 Mc would perhaps have been better with 100 kc guard bands at each end of the spectrum. Similar increasing of the baseband must be done for any final calculations in an actual system.

There are serious practical difficulties with each modulation system considered here. With SSB there are the amplifier linearity and gain instability problems. Intermodulation distortion caused by nonlinearities in gain is a topic beyond the scope of this particular report. However, as has been mentioned elsewhere (see ref. 17), the further development of compandors would alleviate the situation considerably and another development might take the form of more advanced techniques for linearization of the power amplifiers as in ref. 18. Needless to say, the development of power amplifiers in the desired frequency ranges is quite pertinent. It is noted with regard to SSB that a 13.5 dB companding improvement was assumed, whereas 17 or 18 dB improvements are quoted as presently available for 2:1 syllabic compandors.

With FM there is the development problem of practical feedback detectors for 5 Mc or larger baseband frequencies. This is perhaps the most easily solved development problem at present but no published data is available at this time regarding its solution although this is believed to be imminent. Even without feedback detection, the satellite power requirements are not severe so that FM presents one possible solution strictly from an expedient engineering point of view at the present time.

In one respect the information efficiencies and power calculations for PCM are slightly unfair because this system is being compared with FM systems which inherently contain 3 or 4% distortion in the output signal at peak deviation. In other words, the PCM system considered here (10^{-5} error rate) is better quality than the FM systems it is compared with. If we consider the FM system as containing, for example, 3% distortion, the output will contain 97% useful information. On the same basis then, a PCM system for $k = 7$ will contain approximately 1% distortion due to quantization noise and, therefore, an additional 2% can be allowed due to error rate. From ref. 15, an error rate of 10^{-3} yields approximately 1% reduction in output information and a further calculation shows that an error rate of 10^{-2} reduces the information approximately 8%. Therefore, a comparable PCM system could have a $P_e \lesssim 10^{-3}$ or an increase in the basic $(S/N)_c$ ratio, again from ref. 2, of approximately 2 ($\lesssim$ 3 dB). Thus a reduction in PCM transmitter power of 3 dB is perhaps justified in Figs. 6 and 7, with corresponding increase in efficiency,

$$\frac{\log_2 (1 + 10^{1.2})}{\log_2 (1 + 10^{.9})},$$

in Fig. 4.

PCM presents probably the most formidable development problems. First, there is the percent quantization noise increasing with low amplitude signals. A solution to this requires development of compandors similar to those for SSB. However the companding ratios must be much higher and perhaps this represents an impossible problem. In lieu of a solution to the companding problem, the use of PCM might require many more levels than assumed here if the over-all S/N ratio requirements are to be met for low amplitude information signals. Further PCM development is required to achieve practical systems capable of, for example, 77 megabits/sec transmission rate* or perhaps even larger if more than 8 bits/sample are used or more than 1200 channels. This indeed strains the present state-of-the-art with regard to pulse generation, nanosecond switching, and synchronization.

All calculations for signal power assumed the channels to be speech and therefore used speech channel loading factors. Now the question arises: what kind of loading results when music or continuous radio broadcasting is transmitted? Presumably, the peak to rms factor would be reduced in either case so that the present designs would be more than adequate, assuming, of course, that power amplifiers were peak power limited and not rms limited.

No consideration was given to the practical limitations on receiver bandwidth. From this standpoint, as well as conservation of the frequency spectrum, the highest power — lowest bandwidth system is indicated.

There is always the possibility of new modulation schemes arising. Perhaps one of the more interesting is single sideband frequency modulation, see ref. 19. Again, this would have subchannel multiplex problems similar to FM, but the possibility of a 1/2 saving in bandwidth looks attractive. Another recent advance is given in ref. 20 where PPM is used to smooth over the quantization levels of PCM thus requiring less quantization steps for the same S/N ratio and a saving in bandwidth or signal power results.

XIII. CONCLUSIONS

It should be quite apparent that no firm conclusions can be drawn as to which modulation system is best. SSB is the most attractive because of the bandwidth saving, but its use for both up-and-down links

*$2 \times 8 \times 4000 \times 1200 = 76.8 \times 10^6$

depends upon the further development of compandors. At present, the intermodulation problem, while quite important, is perhaps solvable, whereas obtaining sufficient power in the satellite in the near future appears to be out of the question. FM both up-and-down is a feasible system at present. It is a pity that such large bandwidths are required, but at least the initial systems can afford it. Of course, SSB up and FM down is a reasonable alternative at present depending upon sufficient linearity in the SSB power amplifiers. Here, the decision about waiting a short time to utilize a much better system comes to the fore.

With regard to PCM, there appears to be a substantial case against using it at this time. Should feedback detectors become available, the FMFB system is superior to PCM from both power-bandwidth and system complexity standpoints for the S/N ratios required in satellite communications. There is further uncertainty whether 7 or 8 digits/sample is adequate for the low amplitude information signals. Because of satellite weight limitations, one cannot convert to PCM in the satellite; and because PCM cannot trade smaller bandwidth for larger S/N, both up-and-down links must be PCM with larger over-all bandwidth occupancy compared to the SSB up — FMFB down systems. Finally, it appears as though a major breakthrough is required to develop 100 megabit PCM systems. But a note of caution regarding definite conclusions is pointed out because one can rarely predict the extent of scientific development over a several year period with reasonable accuracy.

If one is forced to make a choice at this time, it would seem that SSB up and wideband FM down would be best. One salient reason for this conclusion is the multiple-access capability. Actually, wideband FM both up and down might even be more feasible if all 1200 channels are combined on the ground before transmission to the satellite. Eventually, it would be hoped that SSB for both links will be feasible, but if not, perhaps PCM will be adequately developed for this purpose and FDM subchannel groups can be used. It is important from the satellite communication standpoint that international agreements regarding maximum allowable satellite transmitter power should not preclude the eventual use of SSB for the down link.

One conclusion was that FDM appears to be the multiplex system choice. TDM presents too many practical problems, some beyond the state-of-the-art at present, and does not appear to offer any accompanying advantages. It will require development of new techniques or new systems to invalidate this conclusion.

Finally, a significant conclusion resulting from this study is that the peak power required for SSB and FM is practically constant as n varies from 1 to about 150 channels. The rms power increases with n according to the channel loading, $4 \log_{10} n$, but the peak/rms loading decreases almost as fast. Bandwidth increases directly with n, but a thorough investigation of transmitter costs must be carried out to determine the total variation of transmitter costs with n. It may prove that these costs are primarily determined by peak power rather than rms so that final ground transmitter costs are relatively fixed for $n < 150$ channels.

APPENDIX A

FM IMPROVEMENT FOR FDM SYSTEMS

The problem here is that the (S/N) for each channel is given along with the number of channels, loading factor (channel and peak/rms), and the bandwidth of each channel. From this data we wish to know the (S/N) of the composite wave to be transmitted. In other words, we wish to find out how much the (S/N) per channel is reduced for the composite FDM system including the reduction for FM improvement. From refs. 9, 10 or 15 the FM improvement factor, I_{FM} is given by (1A) (with respect to SSB) for the case of fm, the highest modulating frequency, equaling the baseband.

$$I_{FM} = \frac{3}{2}\left(\frac{\Delta f}{fm}\right)^2 = \frac{3}{2}\beta^2 \tag{1A}$$

where $\left(\frac{\Delta f}{fm}\right)$ if the modulation index. The S/N ratio for the composite FDM wave is given by (2A) where S is increased by L_F and N by n.

$$\left(\frac{S}{N}\right)_c = \left(\frac{S}{N}\right)_{ch} \times \frac{L_F}{n} \tag{2A}$$

where L_F = power loading factor including both channel loading and peak/rms loading. This results from the signal power being increased by the loading and the noise being assumed "white" which means it is simply proportional to bandwidth.

Transmitted S/N, $\frac{C}{N}$, is given simply by dividing (2A) by the over-all improvement factor, (1A), and increasing the noise by the increased bandwidth of the transmitted FM wave.

$$\left(\frac{C}{N}\right) = \left(\frac{S}{N}\right)_{ch} \times \frac{L_F}{n} \times \frac{1}{\frac{3}{2}\left(\frac{\Delta f}{nB_{ch}}\right)^2} \times \frac{nB_{ch}}{BF_M} \tag{3A}$$

where Δf = deviation of the composite FM wave

nB_{ch} = highest modulating frequency

B_{FM} = over-all FM bandwidth

C/N = transmitted S/N ratio

Since the $(\Delta f)^2$ is simply the per channel frequency deviation squared times the power loading factor,

$$\frac{C}{N} = \left(\frac{S}{N}\right)_{ch} \frac{1}{\frac{3}{2}\left(\frac{\Delta f_{ch}}{nB_{ch}}\right)^2 \frac{B_{FM}}{B_{ch}}} \tag{4A}$$

where Δf_{ch} = per channel frequency deviation.
Or in dB

$$\frac{C}{N} = \left(\frac{S}{N}\right)_{ch} \text{db} - 20 \log_{10} \frac{\sqrt{3}\,\Delta f_{ch}}{nB_{ch}} - 10 \log_{10} \frac{B_{FM}}{2B_{ch}} \,. \tag{5A}$$

Actually, the highest modulating frequency might be higher than nB_{ch} because of CCIR channel allocation. Under this condition nB_{ch} must be changed accordingly.

Another equivalent result is obtained by using the composite S/N ratio and the over-all frequency deviation as in (6A)

$$\frac{C}{N} = \left(\frac{S}{N}\right)_{c} \times \frac{1}{\frac{3}{2}\left(\frac{\Delta f}{nB_{ch}}\right)^2 \left(\frac{B_{FM}}{nB_{ch}}\right)} \tag{6A}$$

$$\frac{C}{N}\,\text{dB} = \left(\frac{S}{N}\right)_{c} \text{dB} - 20 \log_{10} \left(\sqrt{\frac{3}{2}}\,\frac{\Delta f}{nB_{ch}}\right) - 10 \log_{10} \frac{B_{FM}}{nB_{ch}}$$

where $(S/N)_c$ dB = $(S/N)_{ch}$ dB + (L_F)dB - 10 $\log_{10} n$.

Actually, (6A) is probably the best formula to use in general because there are times when an over-all modulation index is assumed in the wideband cases and no need arises to calculate the individual channel deviation. Also B_{FM} is a function of Δf and not Δf_{ch}.

Consideration is now given to the FMFB (frequency modulation with feedback detection) system. In this case the intermediate frequency bandwidth of the detector input can be restricted considerably because of the frequency tracking action. Exactly how much the bandwidth can be reduced is a question of practical design limitations. However, from the literature, ref. 11, a figure of three times the baseband appears possible so that value will be used here. What this means is that a receiver improvement factor is now added to the FM system by reducing the received noise.

$$\text{Noise reduction} = \frac{B_{FM}}{3nB_{ch}} = \frac{2}{3}\left(\frac{\Delta f}{nB_{ch}} + 1\right). \tag{7A}$$

The carrier to noise power is, therefore, by dividing (6A) by (7A),

$$\frac{C}{N} = \left(\frac{S}{N}\right)_c \times \frac{1}{2\,\beta^2(\beta + 1)^2} \tag{8A}$$

where β = modulation index = $\dfrac{\Delta f}{nB_{ch}}$, assuming nB_{ch} as the highest modulation frequency, fm. If fm is larger than nB_{ch} then the β term must be changed. It is noted parenthetically here that the reduced bandwidth for the feedback detection can be accounted for in the FM improvement factor as in (8A) in which case P_N calculations would be for the entire B_{FM}. Alternately, the reduced noise bandwidth can be accounted for in the P_N calculation; then the I_{FM} remains the same as without feedback detection in (6A)

Literature on FMFB generally considers the feedback detector as providing a threshold improvement. This results from considering the C/N required at the detector input. Actually, the C/N at the discriminator input remains the same 12 dB as with normal FM detection, but since the feedback tracking changes the wide deviation to narrower deviation, a C/N improvement of several dB takes place between the detector input and the discriminator input. This action effectively reduces the threshold at the detector input. However, for our purposes it appears more straightforward, conceptually, to consider the discriminator input rather than the over-all detector input. Thus 12 dB threshold is assumed in the power calculations, normal I_{FM}, and a reduced noise power used for N(or P_N) in the FMFB case. The reduced power in dB for FMFB is identically equal to the reduction in threshold discussed in the literature.

APPENDIX B

OVERLOAD FACTOR

Various published results show that, as the number of FDM speed channels are increased, the composite becomes more and more like gaussian noise. In fact, the approximation that the composite has a gaussian probability distribution is quite accurate for n, the number of channels, > 64 according to ref. 16. In that reference, experimental values are given for peak to rms ratios where the peak is exceeded 1% and 0.1% of the time. A simple calculation assuming a gaussian distribution of voltages is as follows:

$$P(x) = \frac{2}{\sqrt{2\pi}\,\sigma} \int_x^{\infty} e^{-y^2/2\sigma^2}\, dy \qquad (1B)$$

P (x) is the probability of $y > x$ where σ is the rms value. If we let $\sigma = 1$, the resulting value of x will be the peak voltage to rms voltage for a specified P (x).

Case (1): $$P(x) = 1\% = 0.01 = 1 - \frac{2}{\sqrt{2\pi}} \int_0^{x} e^{-y^2/2}\, dy \qquad (2B)$$

and $$\frac{1}{\sqrt{2\pi}} \int_0^{x} e^{-y^2/2}\, dy = \frac{0.99}{2} = 0.495\ . \qquad (3B)$$

From (3B) and a table of values for the normal curve, we can solve for x.

$$x = 2.575 \qquad (4B)$$

$$\frac{\text{peak power}}{\text{rms power}} = x^2 = 6.62 \text{ or } 8.21 \text{ dB} \qquad (5B)$$

This agrees with Fig. 5.

Case (2): $P(x) = 0.1\% = 0.001$

$$\frac{1}{\sqrt{2\pi}} \int_o^x e^{-y^2/2}\, dy = \frac{0.999}{2} = 0.4995 \qquad (6B)$$

and

$$x = 3.29$$

for which

$$\frac{\text{peak power}}{\text{rms power}} = 10.8 \text{ or } 10.34 \text{ dB} . \qquad (7B)$$

This also agrees with Fig. 5 for the average value of peak/rms ratio. In this report a value of 10.2 dB is used. A number of other papers use a value of 13 dB which is an experimental upper bound from ref. 16, whereas the British Post Office obtained a result of 15 dB.

APPENDIX C

A. COMPARISON OF VARIOUS SPECIFIC RESULTS

Table 1C shows a variety of transmitter powers taken from the literature and compared with results obtained from this report. For the entries taken from this report, the values in Fig. 6 were used except that a satellite receiver noise temperature of 1500°K was assumed and for the earth receiver that of 60°K. This means that the values from Fig. 6 must be increased and decreased by 7 dB respectively for the up and down links.

Differences between the results are not easy to analyze. Presumably, they would all agree if account were taken of the differences in number of channels, S/N ratios, modulation system factors, bandwidths, path losses, etc. However, beyond that, there are apparently some additional differences in the methods of calculation. Needless to say, the final calculations before designing a specific system must be more precise than any of those used in the five sources.

As noted in the Conclusions, SSB up and FMFB down looks quite attractive at this time. FM up is not attractive because of the increased bandwidth; however, if amplifier amplitude linearity is too severe a practical problem, the use of extra bandwidth might be worthwhile. Increased signal processing with FM both up and down is also not very attractive.

TABLE 1C

POWER REQUIREMENT COMPARISON

Source	Channels	System	$(S/N)_{ch}$, dB	Bandwidth, Mc	Peak Power, watts - link
Ghose, ref. 5	1200	FM (wideband)	50	240	141 - down
Pierie, ref. 6	1200	SSB	45	5	100 P (relative power)
	1200	FMFB (β=1)	45	20	100 P (relative power)
	1200	FMFB (β=9)	45	100	P (relative power)
	1200	PCM, 7 digit, AM	45*	100	2 P (relative power)
Firestone, Lutz, and Smith, ref. 7	600	FM (small β)	47	16	1000 - up
	600	FMFB (β=10)	47	86	0.16 - down
Wright and Jolliffe, ref. 8	600	SSB	46	2.5	115 - down
	600	FM	43	50	1.0 - down
	600	PCM, 7 digit, SSB	44	17.5	1.0 - down
Plotkin	1200	SSB	44	4.8	34,800 - up
	1200	SSB	44	4.8	1,380 - down
	1200	FM (β=1)	44	19.2	23,000 - up
	1200	FMFB (β=5)	44	57.6**	9.1 - down
	1200	πPCM 3/2, 7 digit	44	50.4	710 - up
	1200	πPCM 3/2, 7 digit	44	50.4	28 - down

* Reference 6 states that 54 dB peak-peak signal to rms noise was used which converts to 45 dB rms signal to rms noise. However, for 7 digit coding, S/N_q is 44 dB.

** Receiver bandwidth is 14.4 Mc.

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